Extreme movements in Bitcoin prices:

A study based on extreme value theory

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Abstract

This paper studies the extreme movements in Bitcoin prices. Since the introduction of of Bitcoin in 2010, Bitcoin prices have shown dramatic volatility associated with impressive booms and crashes. We use extreme value theory to investigate the statistical distribution of extreme price movements and to compute risk measures commonly used in both risk and asset management by financial institutions. We also draw some conclusions about the status of Bitcoin as a currency or a speculative asset, which is important for practical issues (taxes for example).

Keywords: Bitcoin, extreme value theory, expected shortfall, value at risk, stress testing, volatility, market crashes, currency, speculative asset.

JEL classification: G1, G2

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1 Introduction

Bitcoin was introduced in 2008 by Satoshi Nakamoto in the paper “Bitcoin: A Peer-to-Peer Electronic Cash System”. Since then, the Bitcoin sphere has grown exponentially from an economic point of view; it is also gaining more and more acceptance in society by financial, monetary, and political institution. It opened the door to the world of decentralized cryptocurrencies, but still remains, by far, the cryptocurrency with the most trading and highest market capitalization.

The rise of Bitcoin prices and market capitalization has attracted a growing interest among economists. Previous studies include price formation, price statistical properties.

The objective of this paper is to present some statistical properties of Bitcoin prices focusing on its extreme price movements. Since the introduction of Bitcoin in 2010, Bitcoin prices have shown dramatic volatility associated with impressive booms and crashes. We use extreme value theory to investigate the statistical distribution of extreme price movements and to compute risk measures commonly used in both risk and asset management by financial institutions. We also draw some conclusions about the status of Bitcoin as a currency or a speculative asset.

2 Data and basic statistics

2.1 Data

We have obtained the data used in this study from Bloomberg, with the period of study being October 10, 2010 to August 2, 2016. The price of Bitcoin is the price of the Bitcoin Bloomberg Index (Tracker: BTX). Prices are then used to compute log returns.

2.2 Basic statistics

Before investigating extreme movements in Bitcoin prices, we look at basic statistics as displayed in Table 1. The mean – average daily return over the period – is equal to 0.60%. It reflects the amazing increase in Bitcoin prices since its creation in 2010: the Bitcoin price in dollar rose from 0.10 in 2010 to 568.59 with a minimum of 0.07 and a maximum of 1 137.00. The standard deviation – daily volatility – is equal to 7.18%. This reflects the erratic behavior of Bitcoin prices that we have witnessed since its creation. The skewness is negative and equal to -48.62 indicating an asymmetry in the distribution of returns with more negative extremes than positive extremes. The kurtosis is equal to 745.22 and very high indicating many extreme observations (much more than predicted by the normal distribution whose kurtosis is equal to3).

Table 2 presents the autocorrelation of daily returns for different lags (from 1 day to 20 trading days corresponding to one calendar month). All autocorrelation coefficients are closed to zero, a statistical results consistent with the efficient market hypothesis. Table 4 also presents the autocorrelation of daily squared returns. All autocorrelation coefficients are positive, a fact consistent with ARCH effects.

This preliminary statistical work allows us to draw the following stylized facts about Bitcoin price returns: high positive trend, high volatility, negative skewness, high kurtosis, no autocorrelation for returns, positive autocorrelation for squared returns. Note that such stylized facts are similar to the ones obtained for traditional assets (see for example Cont (2002)).

3 Extreme movements in Bitcoin prices

This part presents results about extreme movements in Bitcoin prices. We use extreme value theory to assess the distribution of extreme returns.

3.1 Extreme value distribution

We implement the peaks-over-threshold method to extract extreme returns: we take a threshold for price returns (defined as percentage points) and we select accordingly all returns that lie above (below) this threshold for positive (negative) extremes. This threshold denoted by θ corresponds to a tail probability *p* of the distribution of returns.

The excess distribution of a random variable *X* over a threshold θ associated to the tail probability p_{tail}, denoted by F_X^{θ}, can be expressed as

$$\boldsymbol{F_X^{\theta}(x) = P(X - \theta \leq x \mid X > \theta) = \frac{F_X(x+\theta) - F_X(\theta)}{1 - F_X(\theta)}}. \tag{1}$$

The distribution of univariate exceedances, F_X^{θ}, can be asymptotically approximated by the generalized Pareto distribution defined by:

$$\boldsymbol{G_X^{\theta}(x) = 1 - p_{tail} \cdot \left(1 + \xi \cdot \frac{(x-\theta)}{\sigma}\right)^{-1/\xi}} \tag{2}$$

where ξ is the tail index and σ is the dispersion parameter.

The tail index is the most important parameter of the extreme value distribution as it measures the weight of the tails of the distribution. With these notations, a positive value for the tail index ξ corresponds to a fat-tailed distribution (Fréchet distribution), a null value to a thin-tailed distribution (Gumbel distribution), and a negative value to a distribution with no tails

(Weibull distribution).

Empirical results are presented in Table 3. A first result is the determination of the threshold value to select extreme observations. For negative extremes, θ is equal to -18.00%, and for positive extremes, θ is equal to 12.65%. For these two values, the tail probability for the left tail and the right tail, p_{tail}, is equal to 1.67%. The scale parameter estimates are similar for both types extremes: 4.24 for the left tail and 5.62 for the right tail. The main difference lies in the tail index: for the left tail, the tail index is equal to 0.34 and statistically different from zero while for the right tail, the tail index is equal to 0.03 and not statistically different from zero. This indicates that there is a heavier tail on the left with many crashes and a thiner tail on the right with few booms.

4 Risk indicators for Bitcoin positions

We consider two measures commonly used in risk management to control positions and in asset management to build portfolios: value-at-risk (VaR) corresponding to a loss occurring with probability *p* and the risk beyond value-at-risk (BVaR) corresponding to the average loss conditional to a loss greater than VaR. The level of risk is given by the probability associated to the VaR or equivalently the average waiting time-period of VaR (the period of time that we have to wait on average to observe a return observation higher than VaR). The risk levels considered cover both ordinary adverse market movements used in risk management based on VaR and extraordinary adverse market movements used for complementary stress testing.[3]

We now use the extreme value distribution to compute risk indicators for positions in Bitcoin. As the extreme value distribution is parametric, it can be used to extrapolate the data. Note that, unlike the extreme value distribution, the historical distribution cannot be used to compute the VaR for high probability values (stress testing) because of the limited number of observations.

Table 4 presents risk indicator for Bitcoin investments based on the quantile of the distribution of extreme returns for both long and short positions. A long position is sensitive to a decrease in Bitcoin prices; risk indicators for a long position then use the left tail of the distribution. Inversely, a short position is sensitive to an increase in Bitcoin prices; risk indicators for a short position then use the right tail of the distribution.

[3] See Longin (2000) for a discussion of the use of VaR and stress testing by financial institutions.

As expected, the VaR is increasing with the probability (or waiting-time period) as we go further in the tails. For a long position, for a low probability (99.6%), the VaR is equal to 31.93% (such a value is exceeded on average every year) ; for a high probability (99.96%), the VaR is equal to 67.90% (such a value is exceed on average every ten years). There is a large difference between these two values indicating that the distribution is fat-tailed on the left. For a short position, for a low probability (99.6%), the VaR is equal to 20.31% (such a value is exceeded on average every year) ; for a high probability (99.96%), the VaR is equal to 30.17% (such a value is exceeded on average every ten years). There is a small difference between these two values indicating that the distribution is thin-tailed on the right.

5 Characterization of Bitcoin

In this section we try to characterize the status of Bitcoin. It was originally presented as a currency or a crypto-currency to be used by economic agents to buy goods and services. Indeed the first transaction was to buy two pizzas at a price of 10 000 bitcoins (worth today more than $100 million!). But does Bitcoin qualify for a currency or is it a speculative asset?

5.1 Bitcoin as a currency?

Money has been around since the Antiquity and has been defined by the Greek philosopher Aristotle in his book *Politics* by its three economic functions: intermediary of exchanges, unit of account, store of value.

As a currency, Bitcoin should be used to buy and sell goods and services. At this time, the use of Bitcoin as an intermediary in the exchanges is quite limited. Few merchants – mainly e-commerce sites referenced on platforms as indicated in CoinDesk (2014) – propose to their customers to pay in Bitcoin but their number seems to increase over time.

As a currency, Bitcoin should be used as a unit of account, that is to say a nominal monetary unit of measure used to represent the real value of any economic item: the displayed price for goods and services in the real world, the value of assets and liabilities in the financial world, the salary amount in job contracts, the rent amount in real estate leases, and the amount of taxes that households have to pay. At this time, it is not the official reference for prices and contracts. Although Bitcoin can sometimes be used as a mean of payment, the price in euros, dollars or other currencies is converted at the time of payment. This situation is certainly explained by the great instability of Bitcoin compared to traditional currencies. As shown in Table 5, the daily volatility for BTC/USD is equal to 7.18%, a number more than ten times higher than the daily volatility for traditional currencies: 0.60% for EUR/USD, 0.60% for JPY/USD and 0.14% for CNY/USD. This is also especially true by looking at the impact of extreme price movements measured by the VaR: considering price Bitcoin price decreases, the long 95% VaR is 9.31% for BTC/USD, compared to 0.98% for EUR/USD, 0.94% for JPY/USD and 0.19% for CNY/USD; and considering price Bitcoin price increases, the short 95% VaR is 10.54% for BTC/USD, compared to 0.98% for EUR/USD, 0.95% for JPY/USD and 0.20% for CNY/USD. Such an extreme volatility prevents for the moment the Bitcoin to be considered as a unit of account.

As a currency, Bitcoin should be used as a store of value. By detaining Bitcoins, an economic agent should be able to transfer his/her purchasing power over time, especially on the short term. Due to its extreme volatility, at the moment, the Bitcoin cannot be considered as a store value. Note that during the financial crisis in Cyprus in March 2013, Bitcoin was considered as a safe haven investment whose availability (by construction peer-to-peer network without a central authority) and liquidity (existence of financial markets to exchange Bitcoins against other currencies) allowed some individuals in Cyprus to circumvent the restrictions imposed during the crisis (no access to deposits, no cash available at ATMs, banks closed for 12 days). In terms of crisis, Bitcoin may be seen as a safe haven investment like gold with more advantages (availability and liquidity).

Considering the three characteristics of money, we conclude that Bitcoin cannot be considered as currency.

5.2 Bitcoin as a speculative asset?

Bitcoin has been considered by many investors as an attractive investment.[4] The market capitalization for Bitcoin has evolved from basically $0.10 at its creation in 2010 to $335 billion on December 27th 2017.

Unlike traditional financial assets (such as stocks and bonds), the Bitcoin does not deliver financial cash flows (such as dividends and interests) that allows to estimate a fundamental value. In other words, Bitcoin has no intrinsic value.[5] Bitcoin must then be considered as a speculative

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[4] Investment in Bitcoin should not be confused with investment in Bitcoin companies (mainly startups) which develop services and applications around the Blockchain technology.

[5] Another point to understand is the relationship between the Bitcoin price and the use of the Bitcoin Blockchain. Given the tremendous investment in Bitcoin start-ups, we can expect that many services and applications will be available in the (near) future. But it is not clear how the value of these side Bitcoin products will relate to the value of the Bitcoin itself.

assets whose value is derived from the confidence placed by investors. When the confidence builds up, the asset price exponentially increases; and then the confidence evaporates, the asset price crashes. History gives us plenty of examples of speculative assets, the first episode being maybe the Tulip mania in the Netherlands in 1637. The price evolution of Bitcoin may then be seen as a succession of speculative bubbles and crashes.

As shown in Table 6, the daily volatility for BTC/USD is equal to 7.18%, a number much higher than the daily volatility for traditional equity indexes: 0.96% for the S&P 500 index, 1.10% for the Eurostoxx, 1.44% for the Nikkei 12225 and 1.64% for the SSE 180. This extreme volatility is also reflected in extreme price movements measured by the VaR: considering price Bitcoin price decreases, the long 95% VaR is 9.31%, compared to 1.57% for the S&P 500 index, 1.77% for the EuroStoxx, 2.25% for the Nikkei 1225 and 2.67% for the SSE 180; considering price Bitcoin price increases, the short 95% VaR is 10.54%, compared to 1.51% for the S&P 500 index, 1.76% for the EuroStoxx, 2.18% for the Nikkei 1225 and 2.43% for the SSE 180. These statistical results highlight the importance of extreme price movements such as booms and crashes in Bitcoin, which is characteristic of speculative bubbles.

The fact that Bitcoin should be considered more as a speculative asset than a currency is important for practical issues such as the tax treatment that should be applied. It suggest that the tax regime for capital gains or losses on Bitcoin should be the same as other financial assets such as stocks.

6 Conclusion

This paper presents an extreme value analysis of Bitcoin prices. From a statistical point of view, it shows that extreme price movements follow a Fréchet distribution with a tail index estimate around 0.30. From an economic point of view, this result shows that Bitcoins exhibits extreme volatility. This paper also sheds light on the nature of the Bitcoin: is it a currency or speculative asset? Our empirical study suggests that Bitcoin should be considered as a speculative asset. This implies that the appropriate tax regime for capital gains or losses on Bitcoin should be the same as other financial assets (stocks for example).

7 References

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Table 1. Basic statistics for Bitcoin daily returns

Moments	Returns (in %)
Mean	0.60%
Standard deviation	7.18%
Annualized standard deviation	137.26%
Variance	0.52%
Skewness	-48.62%
Kurtosis	745.22%

Note: this table gives the basic statistics of Bitcoin daily returns (mean, standard deviation, skewness and kurtosis). Data for Bitcoin prices are obtained from Bloomberg for the period from October 10, 2010 to August 2, 2016.

Table 2. Autocorrelation of Bitcoin daily returns and squared returns

Lag	Returns	Squared returns
1	0.051	0.321
2	-0.010	0.122
3	0.081	0.146
4	0.102	0.082
5	-0.063	0.112
6	-0.001	0.110
7	0.097	0.142
8	0.08	0.100
9	0.026	0.086
10	0.067	0.091
11	0.044	0.063
12	0.058	0.065
13	0.006	0.085
14	-0.040	0.174
15	0.059	0.157
16	0.050	0.036
17	-0.001	0.030
18	0.032	0.042
19	0.050	0.042
20	0.056	0.098

Note: this table gives the autocorrelation of Bitcoin daily returns and squared returns. The estimation period is from October 10, 2010 to August 2, 2016. Data for Bitcoin prices are obtained from Bloomberg.

Table 3. Extreme value statistics for Bitcoin daily returns

	Negative return exceedances	Positive return exceedances
Tail probability	1.67 %	1.67 %
Optimal threshold	-18.00 % (15.71)	12.65 % (10.43)
Scale parameter	4.24 (0.77)	5.62 (0.81)
Tail index	0.34 (0.16)	0.03 (0.11)

Note: this table gives the parameters' estimates of the extreme value distribution for negative and positive daily return exceedances. Standard deviations are given below in parentheses. Data for Bitcoin prices are obtained from Bloomberg for the period from October 10, 2010 to August 2, 2016.

Table 4. Value-at-risk for long and short positions in Bitcoin

	Long position	Short position
99.6% VaR (1 year)	31.93 %	20.31 %
99.92% VaR (5 years)	54.01 %	27.20 %
99.96% VaR (10 years)	67.90 %	30.17 %
99.984% VaR (25 years)	92.04 %	34.11 %
99.992% VaR (50 years)	115.97 %	37.10 %

Note: this table gives the value-at-risk for long and short positions. The level of risk is given by the probability associated to the VaR or equivalently the average waiting time-period of VaR. The risk levels considered cover both ordinary adverse market movements used in risk management and extraordinary adverse market movements used for complementary stress testing. Data for Bitcoin prices are obtained from Bloomberg for the period from October 10, 2010 to August 2, 2016.

Table 5. Basic risk indicators of Bitcoin and major currencies

	BTC/USD	EUR/USD	JPY/USD	CNY/USD
Volatility	7.18%	0.60%	0.60%	0.14%
Long 95% VaR	-9.31%	-0.98%	-0.94%	-0.20%
Short 95% VaR	10.54%	0.98%	0.95%	0.20%

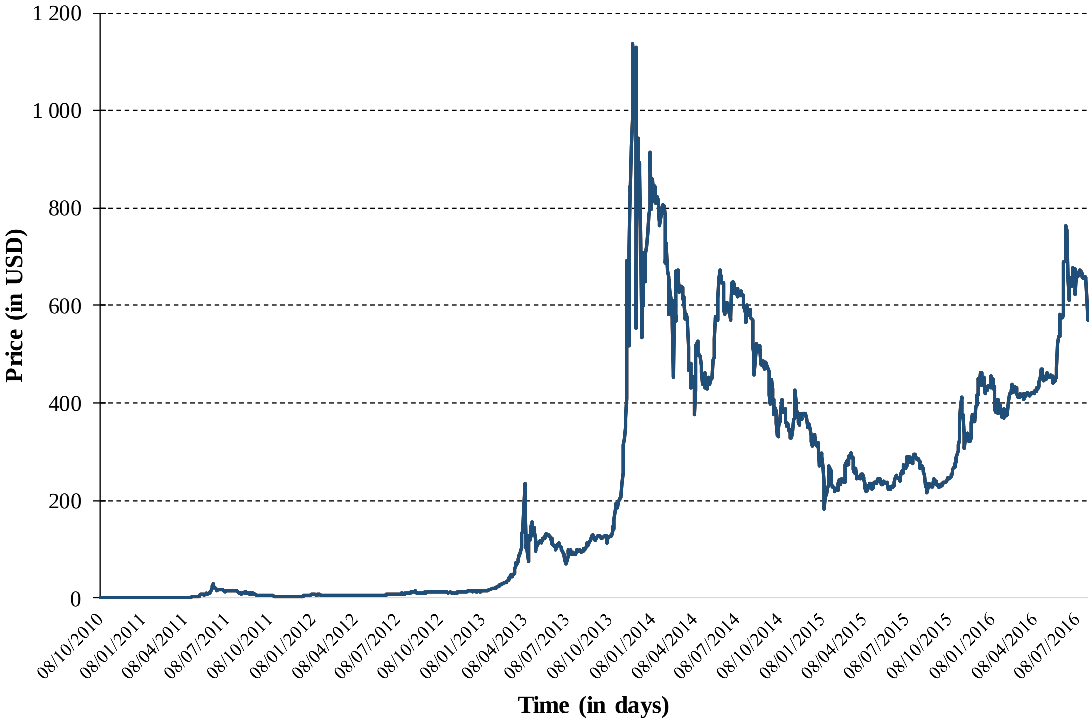
Note: this table gives the basic risk indicators of Bitcoin and major currencies in order to assess the stability of Bitcoin as a currency. Four major currencies are considered: US dollar, Euro, Japanese Yen and Chinese Yuan. Daily volatility and 95% VaR for both long and short positions are computed for each currency against the US Dollar. Daily volatility is computed as the standard deviation of log-returns. VaR is computed with the historical method. Data are obtained from Bloomberg for the period from October 10, 2010 to August 2, 2016.

Table 6. Basic statistics for Bitcoin and major equity indexes

	BIT/USD	S&P 500	EuroStoxx	Nikkei 1225	SSE 180
Volatility	7.18%	0.97%	1.10%	1.44%	1.64%
Long 95% VaR	-9.31%	-1.57%	-1.77%	-2.25%	-2.67%
Short 95% VaR	10.54%	1.52%	1.77%	2.18%	2.43%

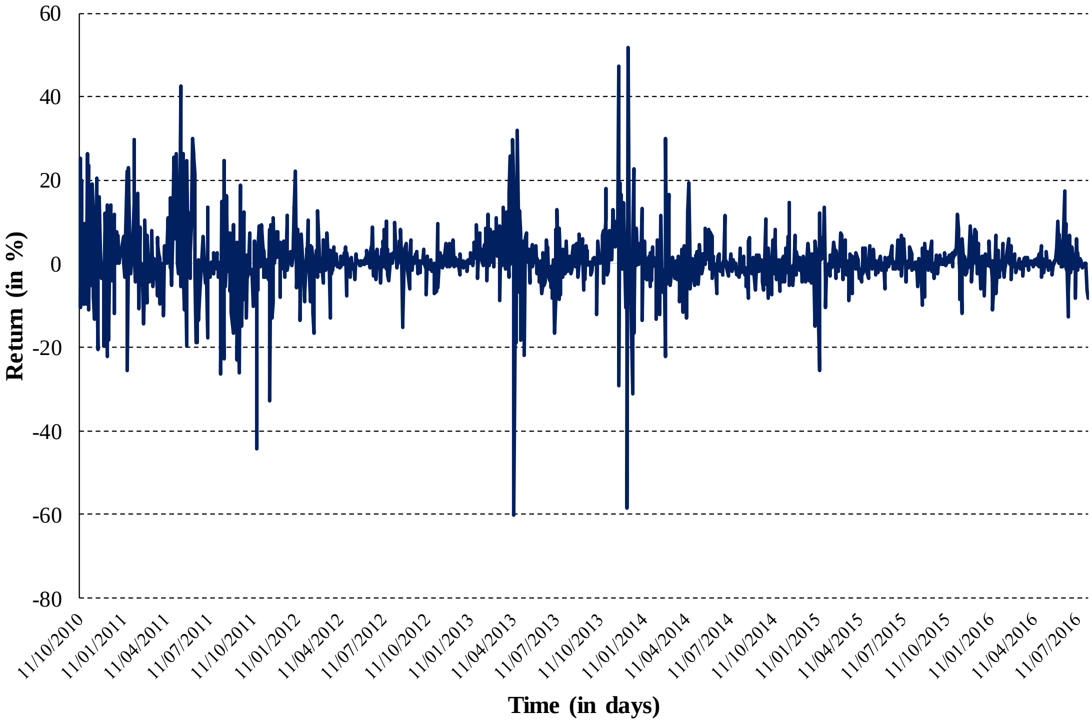
Note: this table gives the basic risk indicators of Bitcoin and major equity indexes in order to. Four major equity indexes are considered: S&P500 index for the US, Eurostoxx for the European Union, Nikkei 1225 for Japan and SSE 180 index for China. Daily volatility and 95% VaR for both long and short positions are computed for each asset class. Daily volatility is computed as the standard deviation of log-returns. VaR is computed for a probability level of 95% with the historical method. Data are obtained from Bloomberg for the period from October 10, 2010 to August 2, 2016.

Figure 1. Evolution of Bitcoin daily price



Note: this figure gives the evolution of the Bitcoin daily price over the period from October 10, 2010 to August 2, 2016. Data for Bitcoin prices are obtained from Bloomberg.

Figure 2. Evolution of Bitcoin daily return



Note: this figure gives the evolution of the Bitcoin daily return over the period from October 10, 2010 to August 2, 2016. Data for Bitcoin prices are obtained from Bloomberg.

Figure 3A. Autocorrelation of Bitcoin daily returns

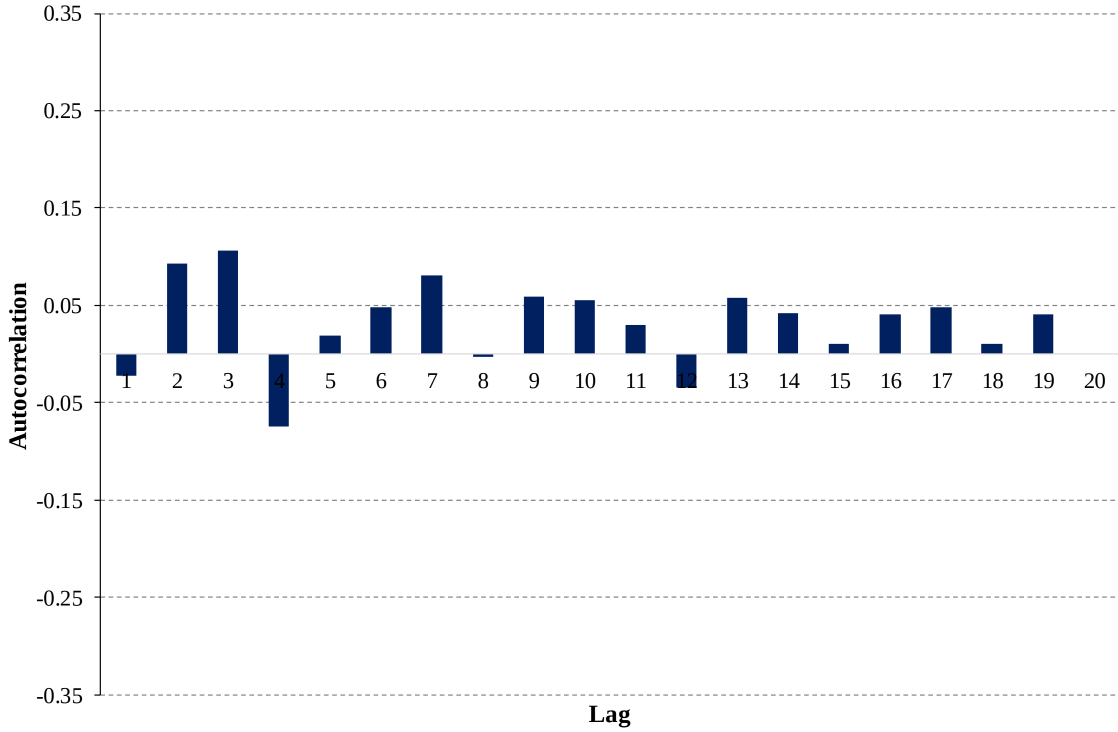
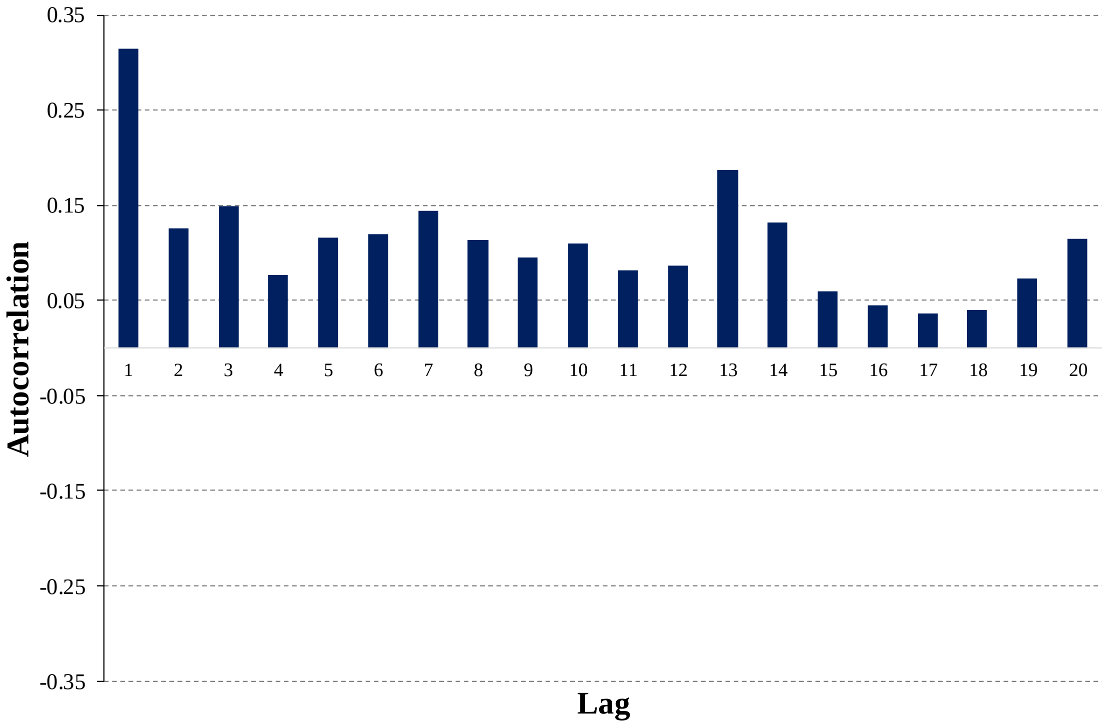


Figure 3B. Autocorrelation of Bitcoin daily squared returns



Note: these figures give the autocorrelation of Bitcoin daily returns (Figure A) and squared returns (Figure B). The estimation period is from October 10, 2010 to August 2, 2016. Data for Bitcoin prices are obtained from Bloomberg.